# Low-frequency dispersion features of a new complex mode for a periodic strip grating on a grounded dielectric slab

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*Abstract* — **A new leaky mode with TE polarization has been found recently [1] on a periodic metallic strip grating placed on a grounded dielectric slab. At low frequencies the dispersion behavior of the fundamental TE mode is completely different when the grating structure is almost closed as compared to when it is almost open, that is, when the ratio of the strip width (*s*) to the grating period (*p*) is close to unity or close to zero. As is discussed here, this difference led to initially puzzling behavior as the ratio *s/p* was varied, and the behavior became clear only after the new complex modal solution was discovered. Results were obtained previously [1] for only one value of dielectric constant and for only two values of *s/p*, leaving many open questions. This paper presents the results of a systematic parametric study which answers most of these questions, and also yields information on what ranges of the parameters can provide values of $\beta/k_0$ and $\alpha/k_0$ suitable for a class of practical leaky-wave antennas.**

## I. INTRODUCTION

This work deals with the analysis of the dispersion features at low frequencies of a basic two-dimensional, periodic, transversely open structure, namely the infinite metal-strip grating on a grounded dielectric slab represented in Fig. 1.

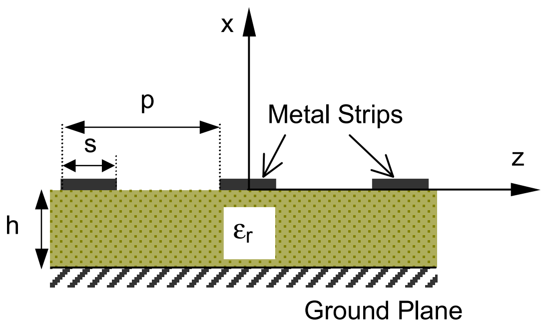



Fig. 1. The periodic open structure treated here: strip grating on a grounded dielectric slab.

In recent years, various leaky-wave antennas derived from this structure have been proposed, based on the radiation of the spatial harmonic of order $n = -1$ of the guided (Bloch) wave excited along their length. For the $n = -1$ space harmonic to provide radiation, the frequency must be above that for the first stop band of the periodic structure. Here we intend to analyze the ***low-frequency*** properties of the fundamental TE mode supported by the metal-strip grating, and the related leakage phenomena due to radiation from the ***fundamental*** ($n = 0$) spatial harmonic rather than the $n = -1$ harmonic.

The existence of a recently discovered new improper complex mode [1] allows us to obtain a consistent view of the low-frequency dispersion features of the fundamental TE mode. Furthermore, this new complex mode explains a mystery, which is described in the next section.

A parametric analysis of the modal behavior as a function of the geometrical and electrical parameters of the structure is presented next, which answers various open questions and also provides information on the possibility of obtaining useful ranges of physical parameters for which efficient radiation from the fundamental harmonic can be obtained and controlled.

## II. THE NEW COMPLEX MODE AND ITS IMPACT ON THE LOW-FREQUENCY DISPERSION FEATURES OF THE FUNDAMENTAL TE MODE

The structure in Fig. 1 has been analyzed by means of the transverse resonance technique, based on a rigorous transverse multimode network representation of the metal-strip grating at the interface between two different dielectrics [1,2].

The aim of our analysis here is to characterize the fundamental TE mode supported by the grating over the full range of values for the *s/p* ratio between the strip width ($s$) and the spatial period ($p$). This mode can be seen either as a perturbation of the $TE_1$ mode supported by a grounded dielectric slab (GDS), the perturbation being represented by the metal strips, or as a perturbation of the $TE_1$ mode of a dielectric-filled parallel-plate waveguide (DPP), the perturbation being represented by the apertures on the upper metallic plate.

The dispersion diagrams (normalized phase $\beta/k_0$ and attenuation $\alpha/k_0$ constants vs. frequency $f$) for these two ***unperturbed*** modes are shown in Fig. 2. Their behaviors below cutoff are very different, since the DPP mode reaches its cutoff when $k_z = 0$, and becomes evanescent for lower frequencies (the dashed line represents its attenuation constant), while the GDS mode reaches its cutoff when $k_z = 1$, and becomes an improper real mode without physical meaning for lower frequencies (the dotted line represents its phase constant).

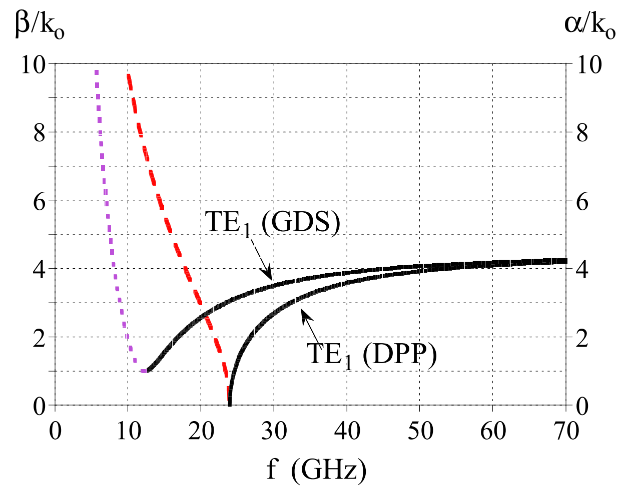



Fig. 2. Dispersion diagrams ($\beta/k_0$ and $\alpha/k_0$ vs. frequency $f$) for the $TE_1$ mode of the grounded dielectric slab (GDS) and for the $TE_1$ mode of the dielectric-filled parallel-plate (DPP) waveguide.

A qualitative change has also been found in the dispersion properties of the mode supported by the grating as the $s/p$ ratio is varied: for a basically closed structure (high values of $s/p$) the bound mode becomes leaky at low frequencies due to radiation from the fundamental spatial harmonic, while for a basically open structure (low values of $s/p$) the proper bound mode becomes improper real at low frequencies.

The Brillouin diagram for a ***basically closed*** structure is shown in Fig. 3(a) (the physical parameters are $s/p = 0.6$, $\varepsilon_r = 20$, $h = 0.140$ cm, $p = 0.338$ cm). In this case the proper bound mode crosses the left side of the triangle at about 22.0 GHz and enters a leaky regime where the $n = 0$ harmonic is improper complex, but physical. This leaky mode is fast, and therefore has physical meaning, down to very low frequencies. The dispersion behavior for this case corresponds to what we would expect.

In the Brillouin diagram of Fig. 3(b) the case is presented for a basically open structure, with $s/p = 0.4$. The other parameters are the same as those in Fig. 3(a). The $n = 0$ space harmonic, shown as a solid line, is proper real and bound inside the triangle. As the frequency is lowered, the solution becomes improper real at the tangency point with the left side of the triangle at 19.56 GHz. The solution remains inside the triangle for lower frequencies (dotted line), but has no physical meaning; at 16.62 GHz it becomes tangent to the right side of the triangle.

We can therefore see from Figs. 3(a) and 3(b) that a leaky mode solution is present outside of the triangle in

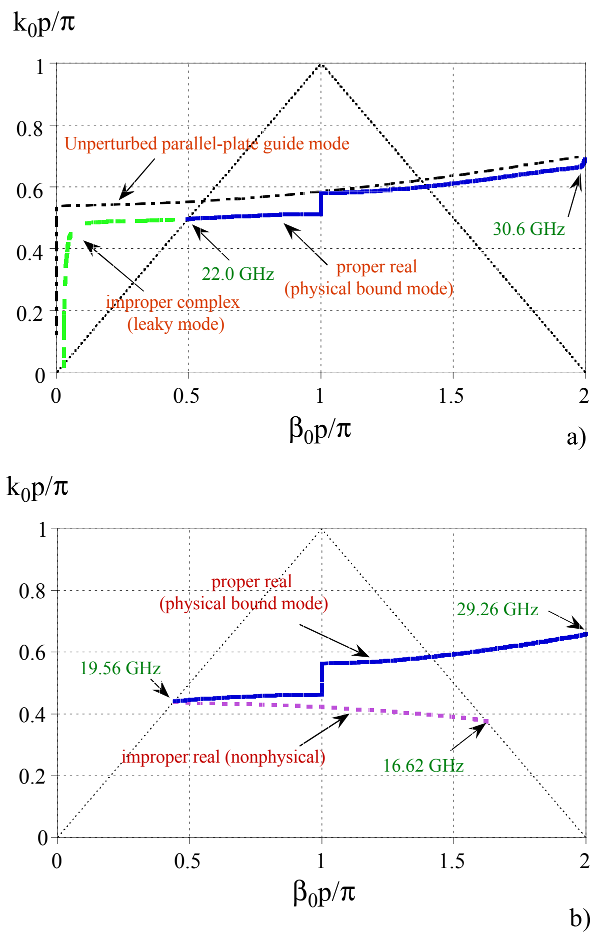



Fig. 3. a) Brillouin diagram ($k_0p/\pi$ vs. $\beta_0p/\pi$) for the periodically-loaded structure of Fig. 1 with $s/p = 0.6$; b) Brillouin diagram for the periodically-loaded structure of Fig. 1 with $s/p = 0.4$. The other parameters are: $\varepsilon_r = 20$, $h = 0.140$ cm, $p = 0.338$ cm.

Fig. 3(a) but not in Fig. 3(b), implying that radiation can occur when $s/p = 0.6$ but not when $s/p = 0.4$. Such behavior seems rather mysterious, particularly since $s/p = 0.4$ corresponds to a more open structure. The mystery is explained, as shown in [1], by the presence of an ***additional, previously unknown, improper complex solution***, which is physical and resembles that of the leaky mode seen in Fig. 3(a) outside of the triangle, but continues inside the triangle at higher frequencies where it is nonphysical.

As will be shown in the next section, this new complex mode exists independently of the fundamental TE mode for low $s/p$ values, whereas it constitutes the low-frequency (complex) continuation of the bound mode solution for high $s/p$ values. The presence of this mode also allows the total solution to maintain the constancy of the algebraic multiplicity of the solutions of the dispersion equation when the frequency and the geometrical parameters are varied continuously.

## III. Parametric Analysis of the New Complex Mode

As part of the study to be described here, we find, as $s/p$ is changed slowly in a continuous fashion, that this new modal solution becomes deformed gradually into the leaky solution shown in Fig. 3(a), and eventually replaces it. The connection between the two complex solutions is accomplished by means of an improper real solution between them. Due to space limitations this evolving process cannot be described here, but it will be discussed during the talk.

In Fig. 4(a), a dispersion diagram of the fundamental harmonic of the new improper complex mode is shown for a grating with spatial period $p = 0.338$ cm on a dielectric substrate with $\varepsilon_r = 20$, $h = 0.140$ cm, for different values of the $s/p$ ratio.

For low $s/p$ values, this new improper complex mode is an independent solution of the dispersion equation, which exists in addition to the fundamental TE mode.

For $s/p = 0.2$ and $\varepsilon_r = 20$, the normalized phase constant $\beta_0/k_0$ is always greater than unity, so that the mode has no physical meaning, and the structure will therefore not leak any physical power. At the frequency for which the phase constant matches that of the improper real solution of the fundamental TE mode, the relevant curve of the attenuation constant changes its slope.

By increasing $s/p$ to 0.4 the mode becomes physical over a wide frequency range, even though its attenuation constant is always very high, with a minimum at the above-mentioned phase-match frequency.

By further increasing the value of the $s/p$ ratio, the minimum of the attenuation constant is lowered and becomes sharper, up to an $s/p$ value above which the complex mode splits into two separate complex branches, with a frequency range between them where the mode is improper real. Under these conditions the complex improper mode is no longer a separate solution, but the lower (and physical) part of it represents instead the low-frequency leaky continuation of the fundamental TE mode (visible in Fig. 3(a) for the case $s/p = 0.6$).

A similar analysis has been performed for a structure with a ***lower*** value of the substrate permittivity. In Fig. 4(b) the dispersion diagram for the new complex mode is shown for the case $\varepsilon_r = 10$ (other parameters as in Fig. 4(a)). The same qualitative behavior already described for the $\varepsilon_r = 20$ case has been found, with a shift of the curves to higher frequency values due to the lowering of the dielectric constant. In this case, for all the values of the $s/p$ ratio shown in the figure, there exists a frequency range in

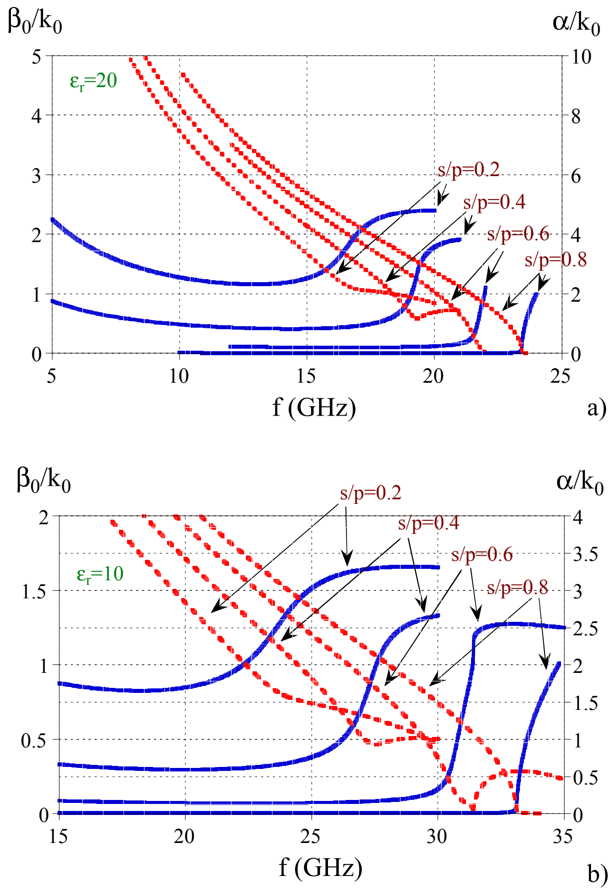



Fig. 4. Dispersion diagram for the new improper complex mode of the periodically-loaded structure of Fig. 1 with different values of $s/p$ ratio: a) $\varepsilon_r = 20$; b) $\varepsilon_r = 10$. Solid lines: phase constants; dashed lines: attenuation constants. *Parameters*: $h = 0.140$ cm, $p = 0.338$ cm.

which the mode is physical. The attenuation constant is very high for structures with low *s/p* values, as already noted, while it tends to zero for structures with high *s/p* values.

The behavior of the new mode for substrates with even lower values of permittivity has been also analyzed. In Fig. 5, a Brillouin diagram is shown for the same structure with $\varepsilon_r$ = 4 and *s/p* = 0.2. A further shift of all the curves to higher frequencies can be observed and, interestingly, in this case the proper TE mode exits the triangle through its tip, with a gap where an improper solution (with $n$ = 0, –1 harmonics improper) acts as a connecting solution. The values of $\alpha/k_0$ are still rather high, but somewhat lower than those shown in Fig. 4.

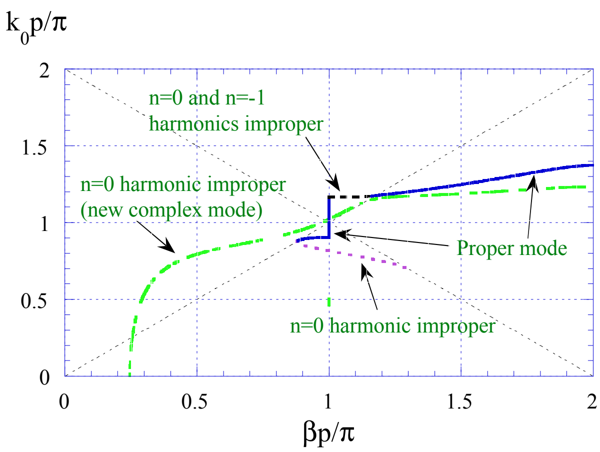
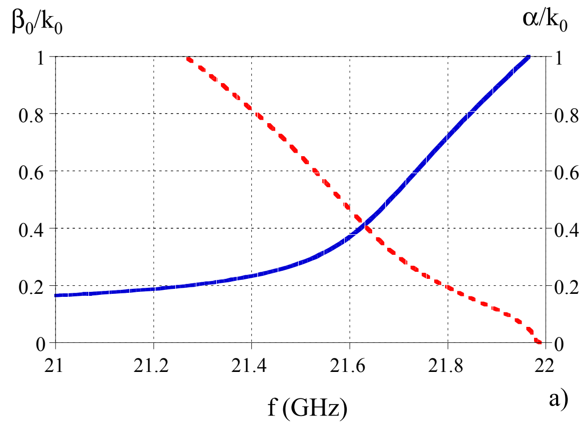



Fig. 5. Brillouin diagram for the structure of Fig. 1 with *s/p*=0.2. *Parameters*: $\varepsilon_r$ = 4, $h$ = 0.140 cm, $p$ = 0.338 cm.

We may also ask if there are parameter ranges for which the values of $\alpha/k_0$ and $\beta/k_0$ are suitable for ***leaky-wave antennas***. It should be noted that, in most cases, especially for low values of the *s/p* ratio, the new complex mode has a very high attenuation constant over the whole frequency range for which the mode is fast (and therefore has physical meaning). Our analysis has led to the identification of cases where efficient radiation through the fundamental harmonic can be achieved by employing basically-closed structures. For suitable choices of the parameters, the attenuation constant can be sufficiently low to permit narrow-beam scan over a wide angular range.

In Fig. 6(a), an enlarged plot of the dispersion diagram for a structure with *s/p* = 0.6 is presented. The attenuation constant rapidly increases as frequency decreases, so that it has rather impractical values unless wide beams are desired. By increasing the value of *s/p* the situation changes, as shown in Fig. 6(b) (*s/p* = 0.8). In this case the



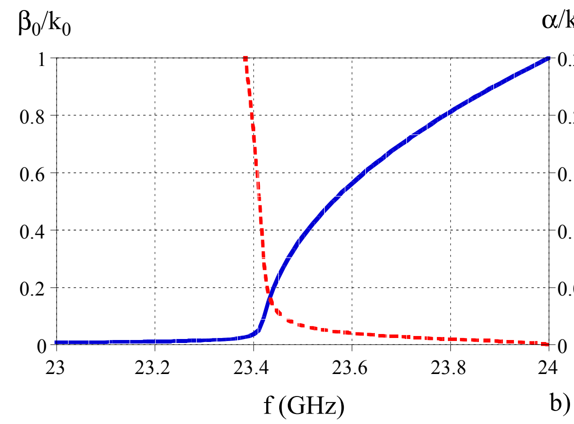




Fig. 6. Dispersion diagram of the new improper complex mode for the structure of Fig. 1 with: a) *s/p* = 0.6; b) *s/p* = 0.8. *Parameters:* $\varepsilon_r$ = 20; $h$ = 0.140 cm, $p$ = 0.338 cm.

curve of the attenuation constant flattens and lowers so that a useful range of frequency values can be obtained where a narrow directive beam can scan in the forward quadrant over a fairly wide angular range.

Additional results have been obtained which cannot be included here but will be presented during the talk.

## References


[1] P. Burghignoli, P. Baccarelli, F. Frezza, A. Galli, P. Lampariello, and A. A. Oliner, "The nature of the radiation at low frequencies from a class of periodic structures," *Proc. 30th European Microwave Conf.*, pp. 349-352, Paris, France, Oct. 2000.

[2] M. Guglielmi and A. A. Oliner, "Multimode network description of a planar periodic metal-strip grating at a dielectric interface - Part I: Rigorous network formulations," *IEEE Trans. Microwave Theory Tech.*, 37, pp. 534-541, Mar. 1989.